Measuring Exponential Growth

Grades 6-8

Math (Pre-Algebra)

Objective

Students reinforce math skills while learning about best practices for protection of water quality in the management of animal feeding operations.

Vocabulary

AFO- animal feeding operation; a place where animals are confined in large numbers and fed
Eutrophication- the depletion of oxygen due to the decomposition of algal bloom, causing death to aquatic life
Exponential growth- growth of populations at increasing rate due to increasing size
Lagoon- an enclosure built to hold liquid waste

Background

Many of the animal products you buy at the supermarket are from animals raised in animal feeding operations, or **AFO**s. These are places where many animals (usually of the same type) are raised in confined situations.

The people who manage AFOs take steps to manage the wastewater from these facilities so it can benefit crops as fertilizer rather than wash into ponds and lakes. Wastewater from AFOs is filled with nutrients from the manure of the animals, as well as anything else washed down the drain (bedding, spilled feed, etc.).

Nutrients that get into ponds and lakes act as fertilizer to the algae in the ponds, just as nutrients on land act as fertilizer to plant life. When too much wastewater from AFOs enters ponds, lakes, or other surface water, the algae multiplies rapidly, covering the surface of the water. This is called an "algal bloom." When the algae die, bacteria in the water decompose the algae. The bacteria experience **exponential growth** to decompose an algal bloom. In the process the oxygen levels in the water decline, killing fish and other aquatic life. This process is called **eutrophication**.

To prevent eutrophication, and to keep the manure nutrients for use as fertilizer for crops, AFOs often have large holding **lagoons** for animal waste. Some AFOs even compost the waste. In cases where many more nutrients are available than the surrounding fields need, some states (such as Oklahoma) have manure transfer programs to move manure to places where it can be used effectively.

Another use for wastewater is as an alternative to petroleum-based natural gas. Anaerobic digesters recover methane from liquid waste, which can be used to produce electricity at the same time that it reduces methane emissions.

Additional Reading

Eberts, Marjorie, *Nature*, McGraw-Hill, 1996.

Farrell, Jeanette, *Invisible Allies: Microbes That Shape Our Lives,* Farrar, Straus and Giroux, 2005.

Fowler, Allan, *If It Weren't for Farmers*, Children's, 1994.

Sayre, April Pulley, *Lake and Pond,* 21st Century, 1997.

Toupin, Laurie, *Freshwater Habitats: Life in Freshwater Ecosystems*, Franklin Watts, 2005.

<u>Activity 1</u> (Math): Measuring Exponential Growth, One 50-minute class period

- Read and discuss background and vocabulary.
- Handout the student worksheet.
- Students will need a scientific calculator.
- Demonstrate the formula that is found on the worksheet to ensure students understand the process.

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Standards

Oklahoma Academic Standards

Activity 1: Measuring Exponential Growth

PA.N.1.1 Develop and apply the properties of integer exponents, including $a^0 = 1$ to generate equivalent numerical and algebraic expressions.

Measuring Exponential Growth

Activity 1: Measuring Exponential Growth

Name: ____

Date: ________________**Class/Hour/Teacher:** _________________

Directions:
Read the information below and then use it to complete the chart.

One way to tell if there is manure contamination of a water supply is to check for the presence of *scherichia coli* bacteria, commonly referred to as *E. coli*. This bacterium is found in the intestines of all animals, including humans, and is an indicator of fecal contamination. Bacteria reproduce in a process called binary fission. The bacterium makes a copy of its one chromosome, grows longer, and then splits in half. Their population doubles with every generation. This is called exponential growth. Some bacteria, like *E. coli*, can divide every 20 minutes.

The number of bacteria present after a certain number of generations is $2^{\text{number of generations}}$ since one cell divides into two each time. So after 15 generations, one bacterium will become 2^{15} = 32,768 bacteria. In other words, one *E. coli* bacteria becomes 32,768 *E. coli* bacteria!

If you want to determine how many bacteria you would end up with if you started out with more than one bacteria, you would just multiply that starting number by the formula above, **starting number of cells X $2^{\text{number of generations}}$**. If you started with 15 *E. coli* bacteria, after five hours (remember one generation for every 20 minutes) you would have 15 X 2^{15} = 491,520 bacteria. Use this information to calculate the following questions.

1. **If you started with one *E. coli* bacteria, how many bacteria cells would there be in each generation for the first 25 generations?**

Gen 1:	Gen 6:	Gen 11:	Gen 16:	Gen 21:
Gen 2:	Gen 7:	Gen 12:	Gen 17:	Gen 22:
Gen 3:	Gen 8:	Gen 13:	Gen 18:	Gen 23:
Gen 4:	Gen 9:	Gen 14:	Gen 19:	Gen 24:
Gen 5:	Gen 10:	Gen 15:	Gen 20:	Gen 25:

2. **Since *E. coli* divide every 20 minutes until optimal conditions have been reached, how long did it take the one *E. coli* to go through 25 generations?**

3. **What kind of graph would best represent the data table in question number 1? Draw a representative graph of that data below.**

Measuring Exponential Growth

Activity 1: Measuring Exponential Growth Answers

Name: ____

Date: ____________________**Class/Hour/Teacher:** ____________________

1. **If you started with one *E. coli* bacteria, how many bacteria cells would there be in each generation for the first 25 generations?**

Gen 1: **2**	Gen 6: **64**	Gen 11: **2,048**	Gen 16: **65,536**	Gen 21: **2,097,157**
Gen 2: **4**	Gen 7: **128**	Gen 12: **4,096**	Gen 17: **131,072**	Gen 22: **4,194,304**
Gen 3: **8**	Gen 8: **256**	Gen 13: **8,192**	Gen 18: **262,144**	Gen 23: **8,388,608**
Gen 4: **16**	Gen 9: **512**	Gen 14: **16,384**	Gen 19: **524,288**	Gen 24: **16,777,216**
Gen 5: **32**	Gen 10: **1,024**	Gen 15: **32,768**	Gen 20: **1,048,576**	Gen 25: **33,554,432**

2. **Since *E. coli* divide every 20 minutes until optimal conditions have been reached, how long did it take the one *E. coli* to go through 25 generations?**

20 minutes per generation / 60 minutes per hour = 3 generations per hour

25 generations / 3 generations per hour = **8 hours, 20 minutes**

3. **What kind of graph would best represent the data table in question number 1? Draw a representative graph of that data below.**

Line graph, graphs may vary