Agricultural Algebra: Pivot Irrigation – Circles in the Field

High School

Geometry and Area

Objectives

Students will calculate the circumference of field irrigation pivots. They will also calculate irrigation rates for multiple pivot scenarios.

Vocabulary

Center pivot irrigation – a method of irrigation, used mainly in the western U.S., in which water is dispersed through a long, segmented arm that revolves about a deep well and convers a circular area from a quarter of a mile to a mile in diameter.
Drip irrigation – a system of crop irrigation involving the controlled delivery of water directly to individual plants through a network of tubes or pipes.
Field capacity – the point at which gravity has drained all the free water down through the root zone, and the remaining water is held by the soil particles
Furrow – a narrow groove made in the ground, especially by a plow
Irrigation – application of water to help plants (especially in agriculture) grow

Background

Irrigation is the number one use of water in Oklahoma. Many of the crops that grow in our state could not survive without an alternative to rainfall and other precipitation. The average annual precipitation for Oklahoma is about 37 inches, but that amount varies a great deal from southeast to northwest and from spring to fall. Annual average precipitation for the eastern part of the state averages between 35 and 57 inches, while the average in the west averages between 15 and 35 inches. Irrigation fills in the gap by providing water stability, which allows Oklahoma farmers to grow a wider diversity of crops.

Crops require a consistent water supply, since 85% of a plant may consist of water. Water moves through and cools the plant as it evaporates while carrying the nutrients needed for its growth. Early water stress limits growth and delays development. Later water stress can result in reduced yields and poor crop quality. Inadequate soil moisture also limits the availability of nutrients needed for root growth.

Water is lost from the soil through evaporation and transpiration. Evaporation is the loss of water by conversion into vapor. Transpiration is the process by which plants give off water vapor through the stomata in their leaves. Evapotranspiration is the loss of water from the soil both by evaporation and transpiration.

Plants can best use water if the soil moisture is maintained near **field capacity** at all times. Field capacity is the point at which gravity has drained all the free water down through the root zone, and the remaining water is held by the soil particles. It is a good balance of soil moisture and aeration.

A crop's water requirement changes during its life cycle. Water requirements are modest during early growth, for most crops. During germination and early growth, soil water content is critical. Once the seed coat ruptures and the radicle (root) and plumule (shoot) emerge, the plant cannot return to seed dormancy. If the germinating seed doesn't receive enough water, it will die. Too much water will displace air containing needed oxygen in the soil, and most plants will suffocate. With increased growth and leaf area, water needs increase. For the many crops harvested as fruits or seeds, meeting water needs during reproduction and grain fill is critical.

Irrigated crops in Oklahoma include corn, hay, wheat, grain sorghum, cotton, peanuts and soybeans. The amount of water needed for irrigating crops depends on many factors, including weather conditions, soil type and the specific needs of the crop. Many Oklahoma farmers plant drought-resistant crops because of the unpredictable nature of our weather. Wheat, hay, cotton and grain sorghum are examples of drought-resistant crops.

Most Plains farmers apply water to their fields using either surface or sprinkler methods. One method makes use of **furrows** plowed between crop rows. Water flows into the furrows from a pipe with holes called "gates" or out of a ditch using tubes called "siphons." Furrow irrigation is used when the land is flat and the soils absorb water slowly.

The leading form of sprinkler irrigation is the **center pivot** method. This sprinkler is a lateral pipe that uses spray nozzles, often suspended on drop tubes and mounted on wheeled structures called "towers." This tower is anchored at the center of the field and automatically rotates in a circle. A center pivot typically has a 1/4-mile radius (approximately 1300 feet) and waters about 130 acres. By adding or subtracting pipeline and towers, producers can irrigate more or less ground. Center pivot systems are useful because they can water soil that absorbs water quickly, and also uneven fields.

Drip irrigation is a method used by some Oklahoma farmers to conserve water, especially for crops that are not drought-tolerant. Drip or trickle irrigation is the frequent application of small quantities of water at low flow rates and pressures. Drip irrigation delivers water to the plant where nearly all the water can be used for plant growth, instead of irrigating the entire field. This method is especially useful in the production of fruit and vegetable crops.

Additional Reading

Groundwater Foundation, *Rainmakers: A Photographic Story of Center Pivots*, Groundwater Foundation, 2005

Activity 1 (Algebra): Circles in the Field, 2-3 50 minute class periods

Materials
- Pen or Pencil
- Calculator
- Activity 1: Center Pivot Irrigation Worksheet

Pivot Irrigation: Circles in the Landscape

High School

Standards

Oklahoma Academic Standards

Activity 1: Circles in the Field

G.2D.1.6	Apply the properties of polygons to solve real-world and mathematical problems involving perimeter and area (e.g., triangles, special quadrilaterals, regular polygons up to 12 sides, composite figures).
G.C.1	Solve real-world and mathematical problems using the properties of circles.
G.C.1.1	Apply the properties of circles to solve problems involving circumference and area, approximate values and in terms of π using algebraic and logical reasoning.

Pivot Irrigation: Circles in the Field

Name: ____

Date: _______________ **Class/Hour/Teacher:** _______________

Define the following terms and write their mathematical equation:

Circumference

Diameter

Radius

1 mile = 5280 feet

Center-pivot irrigation is a method of crop irrigation in which equipment rotates around a pivot. A circular area centered on the pivot is irrigated, often creating a circular pattern in crops when viewed from above. Central pivot irrigation is a form of overhead (sprinkler) irrigation consisting of several segments of pipe (usually galvanized steel or aluminum) joined together and supported by trusses, mounted on wheeled towers with sprinklers positioned along its length. The system moves in a circular pattern and is fed with water from the pivot point at the center of the circle. A full rotation typically occurs every three days. Terrain needs to be reasonably flat, but the system can also function in undulating country. The system is used in parts of the US, Australia, New Zealand, and also in desert areas such as the Sahara and the Middle East. A quarter-mile (1,300-foot) system that irrigates about 120 acres typically costs $325 to $375 per acre, excluding the cost of groundwater well construction, turbine pumps and power units. Longer systems usually cost less on a per-acre basis. For example, half-mile systems (2,600 feet) that irrigate approximately 500 acres cost about $200 to $250 per acre.

(Source: New, Leon, and Guy Fipps, "Center Pivot Irrigation," Texas Agricultural Extension Service, http://itc.tamu.edu/documents/extensionpubs/B6096.pdf)

Problem Set

Mr. Brown owns a section of land that is 1/2 mi X 1/2 mi. He uses this section of land to produce corn to sell. His land is relatively flat, so he is interested in using center-pivot irrigation. He wants to use as much of his land as possible to grow corn. He has hired you to answer the following questions:

1. **What total length of trusses does he need to purchase?**

2. What is the diameter of the irrigated area?

3. What is the circumference of the irrigated area?

4. Draw a sketch of the land showing the irrigation circle.
Label all known mathematical facts.

5. Write a three- to five-sentence paragraph providing your answers to Mr. Brown. Include the cost for the trusses. In addition, answer the following questions in your paragraph:

a. What is the area of irrigation?

b. How much of Mr. Brown's land is NOT being irrigated?

c. What percent of Mr. Brown's land is being irrigated by center-pivot irrigation?

Problem Set 2

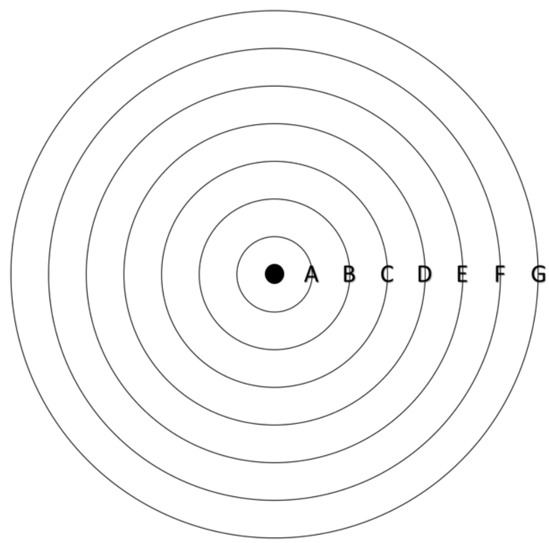
Mr. Brown is so happy with the work that you did for him that he has referred you to his neighbor Mrs. Sloan. Mrs. Sloan grows alfalfa and has noticed that there is too much water close to the center of her field. She asks you to calculate the amount of water being applied by each of the spray nozzles as shown below. She tells you that it takes 72 hours for the pivot to complete a full cycle, and the water application rate is ½ gallon per minute. *Assume the water application rate is the same for each nozzle.*

1. **Why do you think there is more water at the center of the field than the outer field?**

Given: Each wheel line (A, B, C, D, E, F, and G) is spaced 100 feet away from the other wheel lines, (ie., the distance from the center pivot to wheel line D is 400 feet, we did Wheel Line D for you!)

2. **Calculate the circumference of each wheel line.**
   Recall the equation for circumference is:

   Circumference = 2π x radius



A -

B -

C -

D - c= 2π x 400 ft = 6.28 x 400 ft = 2512 ft

E -

F -

G -

3. **Now that you know the circumference for each wheel line, calculate the amount of water being sprayed by each sprayer… remember the sprayers are all set at the same rate, so you only need to do this equation once.**

4. **How many gallons per foot are being sprayed at each wheel line? (Hint: Take the amount of water sprayed in one full cycle, and divide it by the circumference of each wheel line)**

A-

B-

C-

D-

E-

F-

G-

5. **What did you find? Are all wheel lines getting the same amount of water sprayed within the hour? Is the water being sprayed at the same coverage rate at wheel line A as it is at wheel line G? Why or why not?**

Mrs. Sloan realizes the alfalfa at wheel line E is receiving the perfect amount of water. She wonders how she should adjust the spray rate at each sprayer so that the field receives the same amount of water per foot at each wheel line.

6. **Should she change the speed of the pivot or the rate of individual sprayers? Explain your reasoning.**

7. **To reach the optimum spray rate achieved at wheel line E, what should be the rate of water application at each spray line in gallons per foot?**

8. **Using the optimum rate, how much water (in gallons) will be applied at each wheel line in one full cycle?**

A-

B-

C-

D-

E-

F-

G-

9. **Mrs. Sloan decides to purchase adjustable spray nozzles to make sure she can get the right amount water flowing at each wheel line. What is the spray rate in gallons per minute for each wheel line? (hint: use the total gallons per wheel line and the total time for one pivot cycle).**

A-

B-

C-

D-

E-

F-

G-

Extra Credit: Create a scenario similar to the two above to have your fellow classmates try out.

ANSWER KEY

Problem Set 1

Mr. Brown owns a section of land that is 1/2 mi X 1/2 mi. He uses this section of land to produce corn to sell. His land is relatively flat, and so he is interested in using center-pivot irrigation. He wants to use as much of his land as possible to grow corn. He has hired you to answer the following questions:

1. **What total length of trusses does he need to purchase?**

   Since his land is ½ mile square, he will need trusses that equal the radius of the irrigation circle. Since the diameter of the circle will be ½ mile, he will just need a ¼ mile truss which is about 1300 feet.

2. What is the diameter of the irrigated area?
The diameter of the irrigated area is ½ mile or 5280/2 = 2640 feet.

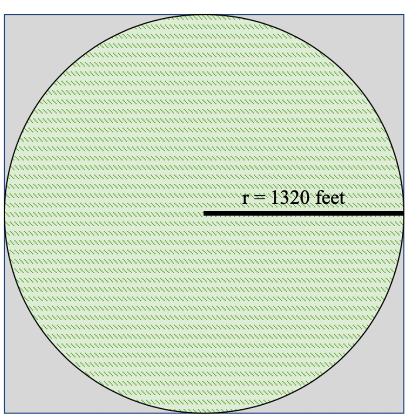
3. What is the circumference of the irrigated area?

Circumference $= 2\pi r$ **or** πd

Circumference $= 2640\pi = 8289.6\ feet$

4. Draw a sketch of the land showing the irrigation circle. Label all known mathematical facts.

Examples include…



Total area of field
2640 x 2640 = 6969600 ft^2

Area of Irrigated space = πr^2 =
5,471,136 ft^2

Non Irrigated Area = 6969600
-5471136
1498464

5. **Write a three- to five-sentence paragraph providing your answers to Mr. Brown. Include the cost for the trusses. In addition, answer the following questions in your paragraph:**
    - **a. What is the area of irrigation?**
    - **b. How much of Mr. Brown's land is NOT being irrigated?**
    - **c. What percent of Mr. Brown's land is being irrigated by center-pivot irrigation?**

Answers will vary. See above.

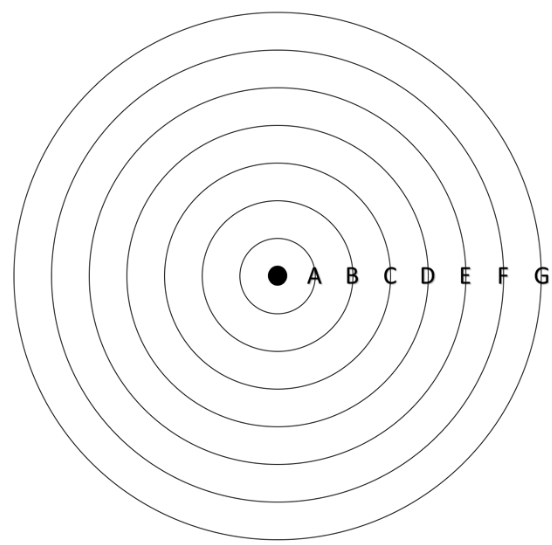
Problem Set 2
Mr. Brown is so happy with the work that you did for him that he has referred you to his neighbor Mrs. Sloan. Mrs. Sloan grows alfalfa and has noticed that there is too much water close to the center of her field. She asks you to calculate the amount of water being applied each of the spray nozzles as shown below. She tells you that it takes **72 hours for the pivot** to complete a full cycle, and the water application rate is ½ gallon per minute. *Assume the water application rate is the same for each nozzle.*

1. **Why do you think there is more water at the center of the field than the outer field?**
Answers will vary, should be something along the lines of same amount of water in a smaller area.

Given: Each wheel line (A, B, C, D, E, F, and G) is spaced 100 feet away from the other wheel lines, (ie., the distance from the center pivot to wheel line D is 400 feet).

2. **Calculate the circumference of each wheel line.**
Recall the equation for circumference is:

Circumference = 2π x radius



A- **c= 2π x 100 ft = 6.28 x 100 ft = 628 ft**

B- **c= 2π x 200 ft = 6.28 x 200 ft = 1256 ft**

C- **c= 2π x 300 ft = 6.28 x 300 ft = 1884 ft**

D- **c= 2π x 400 ft = 6.28 x 400 ft = 2512 ft**

E- **c= 2π x 500 ft = 6.28 x 500 ft = 3140 ft**

F- **c= 2π x 600 ft = 6.28 x 600 ft = 3768 ft**

G- **c= 2π x 700 ft = 6.28 x 700 ft = 4396 ft**

3. **Now that you know the circumference for each wheel line, calculate the amount of water being sprayed by each sprayer during each cycle… remember the sprayers are all set at the same rate, so you only need to do this equation once.**

$$\frac{72\ hours}{pivot\ cycle}\ x\ \frac{60\ minutes}{1\ hour} = 4320\ minutes/pivot\ cycle$$

$$\frac{4320\ minutes}{pivot\ cycle}\ x\ \frac{0.5\ gallon}{1\ minute} = 2160\frac{gallons}{pivot\ cycle}$$

4. **How many gallons per foot are being sprayed at each wheel line? (Hint: Take the amount of water sprayed in one full cycle, and divide it by the circumference of each wheel line)**

A- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{628\ ft}$ **= 3.44 gallons per foot**

B- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{1256\ ft}$ **= 1.72 gallons per foot**

C- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{1884\ ft}$ **= 1.15 gallons per foot**

D- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{2512\ ft}$ **= 0.86 gallons per foot**

E- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{3140\ ft}$ **= 0.69 gallons per foot**

F- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{3768\ ft}$ **= 0.57 gallons per foot**

G- $\frac{2160\ gallons}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4396\ ft}$ **= 0.49 gallons per foot**

5. **What did you find? Are all wheel lines getting the same amount of water sprayed within the hour? Is the water being sprayed at the same coverage rate at wheel line A as it is at wheel line G? Why or why not?**

Each wheel line is spraying the same amount of water, but the lines that are further out have a longer path to travel so there is less water per foot as you go farther out the pivot arm.

Mrs. Sloan realizes the alfalfa at wheel line E is receiving the perfect amount of water. She wonders how she should adjust the spray rate at each sprayer so that the field receives the same amount of water per foot at each wheel line.

6. **Should she change the speed of the pivot or the rate of individual sprayers? Explain your reasoning.**

If she changes the spray rate of all the other sprayers to match the spray rate of wheel line E, then all the field will get the proper amount of water.

7. **To reach the optimum spray rate achieved at wheel line E, what should be the rate of water application at each spray line in gallons per foot?**

   The optimum rate is 0.69 gallons per foot

8. **Using the optimum rate, how much water (in gallons) will be applied at each wheel line in one full cycle?**

A- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{628\ ft}{1\ pivot\ cycle}$ = **433.32 gallons/cycle**

B- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{1256\ ft}{1\ pivot\ cycle}$ = **866.64 gallons/cycle**

C- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{1884\ ft}{1\ pivot\ cycle}$ = **1299.96 gallons/cycle**

D- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{2512\ ft}{1\ pivot\ cycle}$ = **1733.28 gallons/cycle**

E- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{3140\ ft}{1\ pivot\ cycle}$ = **2166.60 gallons/cycle**

F- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{3768\ ft}{1\ pivot\ cycle}$ = **2599.92 gallons/cycle**

G- $\frac{0.69\ gallons}{1\ ft}\ x\ \frac{4396\ ft}{1\ pivot\ cycle}$ = **3033.24 gallons/cycle**

9. **Mrs. Sloan decides to purchase adjustable spray nozzles to make sure she can get the right amount water flowing at each wheel line. What is the spray rate in gallons per minute for each wheel line?**

A- $\frac{433.32\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.10 gallons/minute**

B- $\frac{866.64\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.20 gallons/minute**

C- $\frac{1299.96\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.30 gallons/minute**

D- $\frac{1733.28\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.40 gallons/minute**

E- $\frac{2166.60\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.50 gallons/minute**

F- $\frac{2599.92\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.60 gallons/minute**

G- $\frac{3033.24\ \text{gallons}}{1\ pivot\ cycle}\ x\ \frac{1\ pivot\ cycle}{4320\ minutes}$ = **0.70 gallons/minute**

Extra Credit: Create a scenario similar to the two above to have your fellow classmates try out.

Answers will vary